

- 35 [7].—HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Toroidal Harmonics, III: Functions of the First Kind—Orders 0—10*, Report ARL 70-0127, Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, July 1970, iv + 391 pp., 28 cm. [Copies obtainable from National Technical Information Service, Springfield, Virginia 22151. Price \$3.00.]

The first table in this report consists of 11S values (in floating-point form) of the Legendre function of the first kind,  $P_{n-1/2}^m(s)$ , for  $m = 0(1)10$ ,  $s = 1.1(0.1)10$ , and degree  $n$  ranging from 35 to 160, as in two earlier companion reports [1], [2], which were devoted to the tabulation of the Legendre function of second kind,  $Q_{n-1/2}^m(s)$ .

This table is followed by a tabulation, also to 11S, of the same function for similar orders  $m$  and for arguments  $s = \cosh \eta$ , where  $\eta = 0.1(0.1)3$ . The upper limit for the degree,  $n$ , here varies from 34 to 450.

A concluding table gives values of the cross product  $P_{n+1/2}^m(s)Q_{n-1/2}^m(s) - Q_{n+1/2}^m(s)P_{n-1/2}^m(s)$  to 16S for  $m = 0(1)10$ ,  $n = 0(1)450$ . This table evolved from spot-checking the other tables by means of identities that were derived from the known Wronskian relation and that are presented in the introductory section describing the method [3] of calculation by means of IBM 1620 and IBM 7094 systems.

Also included is a discussion of the application of toroidal functions to the determination of the potential field induced by a charged circular torus.

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Toroidal Harmonics, I: Orders 0—5, All Significant Degrees*, Report ARL 69-0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math. Comp.*, v. 24, 1970, pp. 489–490, RMT 36.)

2. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Toroidal Harmonics, II: Orders 5—10, All Significant Degrees*, Report ARL 69-0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969. (See *Math. Comp.*, v. 24, 1970, pp. 989–990, RMT 70.)

3. HENRY E. FETTIS, "A new method of computing toroidal harmonics," *Math. Comp.*, v. 24, 1970, pp. 667–670.

- 36 [8].—LUDO K. FREVEL, *Evaluation of the Generalized Binomial Density Function*, Department of Chemistry, The Johns Hopkins University, Baltimore, Maryland, 1972. Ms. of 13 pp. deposited in the UMT file.

The author defines herein a generalized binomial density function by the relation

$$\beta(x; n, \alpha) = \frac{\Gamma(1 + 2n)(\sin \alpha)^{2(n+x)}(\cos \alpha)^{2(n-x)}}{\Gamma(1 + n + x)\Gamma(1 + n - x)}$$

which reduces to the standard binomial function  $b(k; m, p)$  when  $x = m/2 - k$ ,  $n = m/2$ , and  $\alpha = \arcsin p^{1/2}$ .

A table of this function is included for  $\alpha = \pi/4$ ,  $x = 0(0.05)3$ , and  $n = -0.1, 0, 0.1, 1, 2$ ; it was computed to 10D on a Wang 360 calculator before truncation of the final tabular entries to 8D.

In addition, a probability density function  $\phi_n(x)$  is defined in terms of  $\beta(x; n, \alpha)$  by the relation

$$\phi_n(x) = \left[ \int_{-n-1}^{n+1} \beta(\xi; n, \alpha) d\xi \right]^{-1} \left[ \frac{1}{2} + \frac{1+n-|x|}{2|1+n-|x||} \right] \beta(x; n, \alpha),$$

and the normalizing factor  $\int_{-n-1}^{n+1} \beta(\xi; n, \alpha) d\xi$  is tabulated to 5D for  $n = 0, 0.1, 0.5, 1, 2, \infty$ .

Two computer plots are also included: one of  $\beta(x; n, \pi/4)$  for the tabular arguments; the other of  $\beta(x; 0, \alpha)$  for  $\alpha/\pi = 0.05(0.05)0.25$  and  $-1 \leq x \leq 3$ .

J. W. W.

37 [9].—M. LAL, C. ELDRIDGE & P. GILLARD, *Solutions of  $\sigma(n) = \sigma(n+k)$* , Memorial University of Newfoundland, May 1972. Plastic bound set of 88 computer sheets (unnumbered) deposited in the UMT file.

The function  $\sigma(n)$  is the sum of all positive divisors of  $n$ . Table 2 contains 50 separate tables. The  $k$ th of these gives all  $n \leq 10^5$  such that

$$(1) \sigma(n) = \sigma(n+k).$$

Also listed here are  $n+k$  and  $\sigma(n)$ .

Table 1 gives the number of solutions above for each  $k$ . Thus,  $k=1$  has 24 solutions, the first being  $n=14$  and the last being  $n=92685$ .

An earlier table, apparently unpublished, was by John L. Hunsucker, Jack Nebb, and Robert E. Stearns, Jr. of the University of Georgia. This larger table listed all 113 solutions for  $k=1$  and  $n \leq 10^7$ . Their last is  $n=9693818$ . They had the same 24 solutions  $< 10^5$ . They also computed (1) for all  $1 \leq k \leq 5000$  and  $n+k \leq 2 \cdot 10^5$ , and so should include everything here deposited. I have not seen this larger table.

In their larger range of  $n$  there are still only two solutions for  $k=15$ :  $n=26$  and  $n=62$ . Won't someone please prove that there are only two? Or are there others?

D. S.

38 [9].—SOL WEINTRAUB, *Four Tables Concerning the Distribution of Primes*, 23 pages of computer output deposited in the UMT file, 1972.

Tables 2, 2A and 2B (6 pages each) are very similar to Weintraub's earlier [1]. See that review for the definitions of GAPS, PAIRS, ACTUAL, and THEORY. For the same variable  $k=2(2)600$ , Table 2 lists these four quantities for the 11078937 primes in  $0 < p < 2 \cdot 10^8$ ; Table 2A for the (unstated number of) primes in  $10^{16} < p < 10^{16} + 25 \cdot 10^5$ ; and Table 2B for the 255085 primes in  $10^{17} < p < 10^{17} + 10^7$ . Nothing extraordinary occurs in these tables that requires special mention. The largest gap here is a case of  $k=432$  in Table 2A. ACTUAL and THEORY agree very well, as expected.

Table A (5 pages) covers the same range as Table 2 does. For  $n=1(1)200$  it first lists